A Nondegenerate Traveling-Wave Parametric Amplifier*

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Summary—**The traveling-wave parametric amplifiers reported to date are not sufficiently competitive to ensure their use in advanced radar systems. The major hindrance is the relatively high-radar noise figure which is due to the fact that the signal and idler frequencies are about equal; *i.e.*, the amplifier is degenerate. Initial experiments directed toward obtaining a nondegenerate microwave traveling-wave parametric amplifier are reported in this paper. A promising circuit has been developed and an *S*-band nondegenerate amplifier has been built and tested. In the first part of the paper the circuit is described and the experimental results are given. The second part of the paper describes procedures which have proved useful in the development of the circuit and which should also prove useful in future investigations.**

Introduction

THE TRAVELING-WAVE parametric amplifier (TWPA) offers the advantage of increased bandwidth over the single diode parametric amplifier. This is still true in spite of recent advances in broad-band single diode amplifiers. Assuming that the TWPA will continue to have a bandwidth advantage over the single diode PA, the major competitive device is the low-noise traveling-wave tube (TWT). At present, low noise TWT's have noise figures of about 4 to 5 db at *S* band, although somewhat lower values have been reported. TWPA's reported at *S* band also have single channel noise figures of about 4 to 5 db, so the two devices are about equal in the noise figure race. However, the low noise TWT has greater bandwidth, gain, and actually a lower theoretical noise figure than present TWPA's. The TWPA does in turn have some advantages over the TWT, such as being a solid-state device and hence requiring no processing, vacuum, or magnets; also the power supplies are less complex and, of course, there are some applications where the much lower double channel TWPA noise figure can be used. But for most purposes, the advantages the TWPA offers are not so important as those offered by the TWT. It is obvious that if the TWPA is to survive and have a sizable market, the noise figure must be considerably lowered.

The noise figure can be lowered by making the amplifier nondegenerate, *i.e.*, by making the idler frequency much higher than the signal frequency. This requires either a separate circuit or a higher order pass band of the signal circuit to support the idler. The use of a separate circuit offers greater flexibility for the amplifier, but also would probably lead to a large, cumbersome device. In order to obtain not only a low noise amplifier, but also a compact and practical system component, the amplifier reported in this paper uses a higher order passband to support the idler. Furthermore, another higher order pass band is used to support the pump, although it would be possible to feed the pump separately to each diode if a suitable pump pass band were unavailable. The reason for avoiding a parallel feed pump was that experience has shown that a propagating pump requires less power and also makes possible a much simpler structure.

Part I—Circuit and Experiments

There are four basic electrical requirements which the circuit must meet to be suitable for a nondegenerate TWPA using separate pass bands for the signal, idler, and pump. The first two requirements are the usual frequency and phase conditions as given by

$$\omega_s + \omega_i = \omega_p, \tag{1}$$

$$\theta_s + \theta_i \approx \theta_p. \tag{2}$$

The third condition is that the Brillouin diagrams (ω-β curves) of the signal and idler pass bands be similar in shape and approximately equal in size with respect to frequency; this will mean that (1) and (2) will be satisfied over a wide frequency range. The fourth condition is that the interaction impedance be high for all three pass bands. The concept of interaction impedance, as applied to a filter type TWPA, is discussed in the literature[1,2] and will also be treated in somewhat more detail later. For present purposes, it is sufficient to know that the interaction impedance is a measure of the strength of coupling between the diodes and the fields of the circuit; a high impedance means strong coupling while a low impedance means weak coupling. It is apparent that three pass bands of some filter circuits could be found which satisfy the first three conditions, but with one or more of the pass bands having very low interaction impedance, thus making amplification impossible or possible only with excessive pump power.

A circuit has been developed to meet the above requirements and is shown schematically in Fig. 1. The circuit consists of six inductively coupled coaxial cavi-

* Received by the PGMTT, August 21, 1961; revised manuscript received, October 9, 1961. This paper was presented at WESCON, August 22–25, 1961.

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[1] K. P. Grabowski and R. D. Weglein, "Coupled-cavity traveling-wave parametric amplifiers: Part II—Experiments," PROC. IRE, vol. 48, pp. 1973–1987; December, 1960.

[2] M. R. Currie and R. W. Gould, "Coupled-cavity traveling-wave parametric amplifiers: Part I—Analysis," PROC. IRE, vol. 48, pp. 1960–1973; December, 1960.

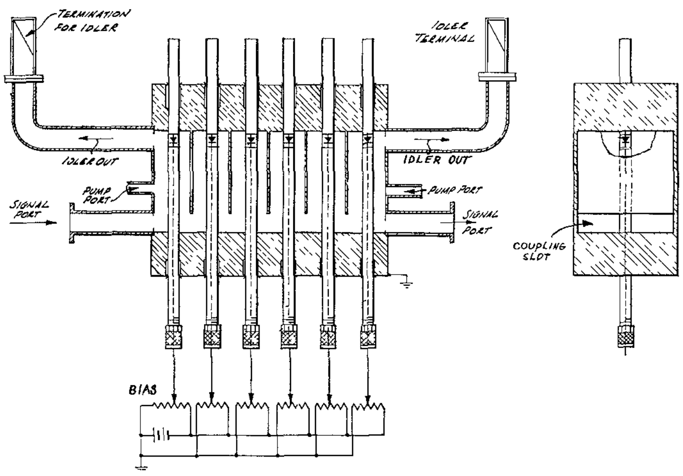

Fig. 1—Schematic drawing of the nondegenerate TWPA.

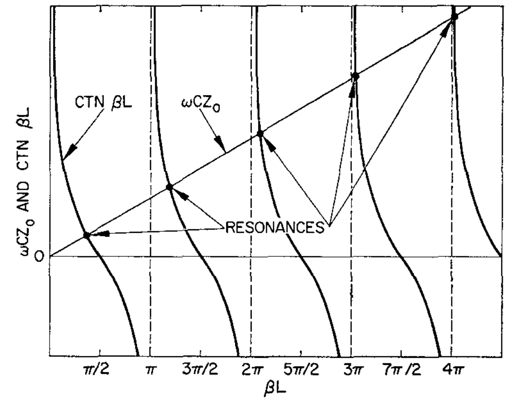

Fig. 2—Solution of the equation for the resonant frequencies of a single cavity.

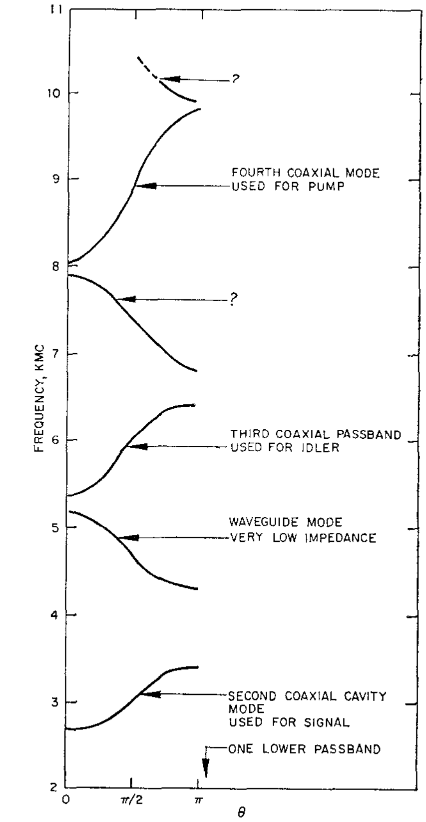

Fig. 3—Pass bands (ω-β curves) for the nondegenerate TWPA.

ties with a diode terminating each cavity at the extremity farthest from the coupling slot. The pass bands of the circuit can be understood by considering the resonant frequencies of a single cavity.[3] Each cavity, neglecting the coupling hole, is simply a length of coaxial line shorted at one end and capacitively loaded by the diode at the other end. Considering the diode as a pure capacitance, this cavity is resonant whenever the familiar equation (3) is satisfied.

$$\omega C Z_0 = \operatorname{ctn} \beta L, \tag{3}$$

where

C is the diode capacitance,
Z_0 is the coaxial line characteristic impedance,
L is the cavity length, and $\beta = \omega/v$, where v is the velocity of light.

The solution of (3) is shown in Fig. 2. The lowest resonant frequency can be seen to occur with L less than a quarter wavelength; the second, third and fourth resonances occur, respectively, at frequencies such that L is slightly greater than one-half, one, and three-halves wavelengths. The latter three resonances are of major interest since the sum of the frequencies of the second and third resonance is approximately equal to the frequency of the fourth resonance. The field pattern of all the modes is such that the magnetic field is a maximum at the end of the cavity away from the diode. Therefore, a slot between the cavities at the extremity opposite the diode will provide inductive coupling for all of the modes. As coupling is provided, pass bands of the circuit occur about the resonant frequencies with the bandwidth proportional to the strength of coupling. For the circuit of Fig. 1, the pass bands shown in Fig. 3 were measured for coupling slots extending completely across the broad face of the cavities. The lowest frequency pass band is not shown in Fig. 3 since at present it is of little interest. The extra pass bands shown in Fig. 3 are associated with resonances of the cavity without the center conductor, *i.e.*, with the lengths of waveguide forming the outer conductor of the cavity. These waveguide modes couple very weakly to the diodes and do not affect the operation.

The circuit was first operated at S band as a degenerate amplifier. A partially disassembled degenerate X-band amplifier is shown in Fig. 4. This circuit is a direct scaling of the S-band model. In the degenerate amplifier the second coaxial pass band is used for both

[3] The approach to finding the cavity resonant frequencies given here is oversimplified though essentially correct. A more detailed treatment is given by K. P. Grabowski, "Circuits for Traveling-Wave Parametric Amplifiers," Hughes Research Labs., Malibu, Calif., Res. Rept. No. 175; December, 1960.

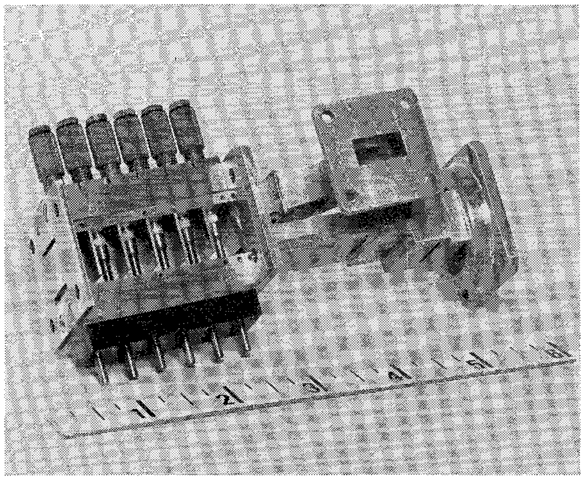

Fig. 4—Degenerate amplifier at X band.

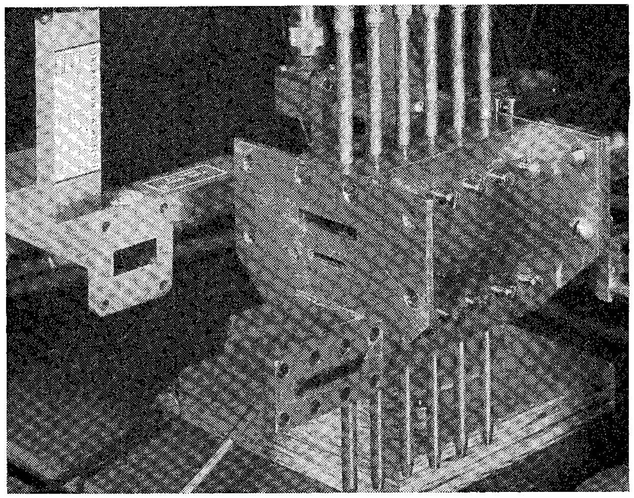

Fig. 5—Nondegenerate S-band TWPA.

the signal and idler and the pump propagates along the third coaxial pass band. The signal input and output are by means of slots similar to the coupling slots between the individual cavities; the pump ports use a similar slot at the opposite end of the cavity. Filters are provided at the various ports to block the unwanted frequencies. Results obtained at S band with the degenerate amplifier include 13-db $\pm$1-db insertion gain over 500-Mc bandwidth and a radiometer (double-channel) noise figure of 1.5 to 2 db. From the experiments with the degenerate amplifier, it appears that bandwidths of the order of 50 per cent are obtainable with coupling slots at both ends of the cavities.

The nondegenerate amplifier was obtained from the degenerate amplifier simply by rearranging the input and output ports, changing the diodes, and rematching the ports for the signal and idler. The nondegenerate S-band amplifier with the input waveguides removed is shown in Fig. 5. (To avoid confusion, it should be noted that the amplifier of Fig. 5 is upside down with respect to the amplifier of Fig. 4.) The guide at the bottom goes to the idler termination. The top slot is for the signal and the center slot is for the pump.

The experimental results for the nondegenerate amplifier will now be summarized. The amplifier was first operated with Hughes Aircraft Company's gold-bonded germanium diodes since the degenerate amplifier had operated well with these diodes. However the low Q of these diodes at the idler frequency (6 kMc) and at the pump frequency (9 kMc) limited the gain to only 2 to 3 db with up to 3 watts of pump power. Next, six Microwaves Associates' MA450F diodes were tried with considerably better, but not yet good, results. Some gain curves taken with the MA450F diodes are shown in Fig. 6. The pump power was still high and the radar noise figure was 7 db, which is quite poor. The low Q of the diodes was again blamed for the poor results, although the fact that the pass bands did not occur with exactly the desired frequency and phase characteristics

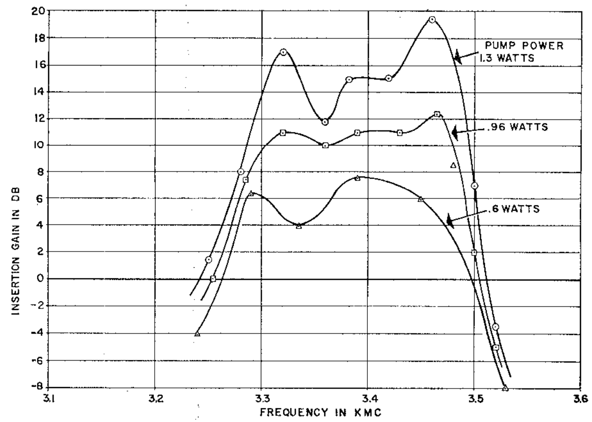

Fig. 6—Experimental results with 6-MA450F diodes.

also degraded the performance. Next, a combination of three MA450G diodes and three Hughes-made silicon mesa diodes with similar characteristics were used. Very encouraging results were obtained; gain of greater than 10 db over almost 400 Mc was measured together with a radar noise figure of less than 4 db. The pump power required was about 300 Mw. Unfortunately, the failure of two of the diodes prevented extensive tests and it has not yet been determined exactly why this combination of diodes gave such improved results.

The following conclusions can be drawn from the experiments performed to date: The feasibility of broadband nondegenerate TWPA operation has been demonstrated. The circuit performs essentially as designed when the correct diodes are supplied. It is not so important that the diodes be closely matched to each other, but rather that the diodes have the value of capacitance required by the circuit to align the three pass bands. Slight differences in the capacitance from diode to diode can be compensated by changes in the dc bias and performance is not seriously degraded. When in operation the amplifier was quite stable and results were reasonably reproducible.

Part II—Design Considerations

There have been several theoretical investigations of TWPA's. The theoretical work generally applies to both degenerate and nondegenerate amplifiers and starts with the assumption that circuits are available having optimum or near optimum characteristics. The theories show that if (1) and (2) are satisfied, exponential gain is possible and some general characteristics of the gain are usually given. But essentially no information is obtained to aid the designer in developing a useful circuit at microwave frequencies. It is the purpose of this part of the paper to discuss the circuit conditions necessary for TWPA operation and to show how to simply determine whether or not a particular circuit is suitable. Four conditions for nondegenerate TWPA's amplification were given above; whether or not a potential circuit is suitable in light of these conditions can be determined from the Brillouin diagram of the pass bands and the interaction impedance. The evaluation of a circuit in terms of the Brillouin diagram (ω-β curve) and the interaction impedance (K) is to a certain extent independent of the input-output matches and the particular diodes.

Consider first the ω-β curve; its use for degenerate amplifiers was pointed out in the literature,[1,2] and the interpretation for nondegenerate amplification is essentially the same. The first step in evaluating a possible circuit is to determine the ω-β curves for the signal, idler and pump pass bands, either analytically or experimentally. A pump frequency, f_p is then selected along with the pump phase shift θ_p as determined by the pump ω-β curve. Points of the ω-β curve of the idler are subtracted from the pump frequency and phase to obtain a generated signal curve, *i.e.*, (1) and (2) are solved for the signal frequency and phase using the selected pump parameters and the idler ω-β curve. Similarly a generated idler curve can be obtained. The significance of the generated modes can be interpreted in terms of coupled mode theory or may be simply interpreted as showing graphically how well (1) and (2) are satisfied and in turn how well the first three requirements for nondegenerate TWPA operation are fulfilled. As an example, Fig. 7 shows the ω-β curves for the idler and signal pass bands of the coupled coaxial cavity circuit for a particular set of diodes and a particular size of the coupling hole between the diodes. The pump pass band is not shown, but the pump phase and frequency are indicated and the generated signal pass band is shown dashed. Interpreting the dashed and solid curve at the signal frequency as two modes coupled by the pump, gain is to be expected where the two curves are coincident or close to each other. Several characteristics of the expected response are immediately evident. The right half of the ω-β curve represents, by definition, the forward direction and since the curves on the right are close over a considerable frequency range a fairly broad-band response can be expected. However, since the idler pass band is wider than the signal pass band, the curves are close together only over about a quarter of the available signal band and wider bandwidth would be possible with a narrower idler band. The left half of the curve represents the reverse direction through the amplifier and a wide separation is desired for low reverse gain. But notice that there is coincidence on the left for a narrow band of frequencies indicating a potential point of instability although the low forward gain in this range will probably prevent oscillation. Other values of pump frequency and phase can be chosen depending upon the pump ω-β curve and the effect of such a change will be to shift the dashed curve with respect to the solid. It is quite easy to determine the optimum pump values by drawing the dashed curve on separate transparent paper and moving it on top of the solid curve to obtain coincidence over the largest frequency range in the forward direction and to obtain farthest separation in the reverse; this movement must of course be restricted by the range of pump values that are possible.

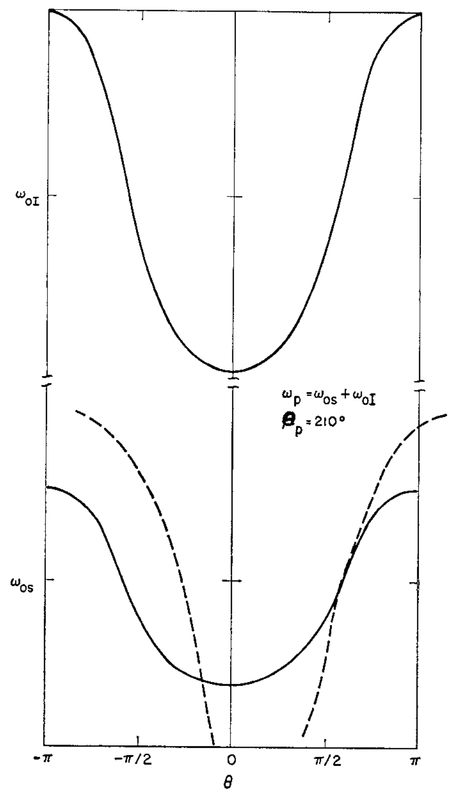

Fig. 7—Nondegenerate TWPA ω-β curves showing the generated signal mode.

The previous example illustrated a method of determining whether the pass bands are properly aligned, approximately the variation of gain with frequency, and the characteristics of the reverse gain. The example was typical of situations that occur with the circuit presented in this paper, other circuits could differ widely in the shape of the ω-β curve, but the method is still valid. It must be emphasized that the amount of gain is not predicted by the Brillouin diagram but the variation of gain with frequency in the forward and reverse direction, the bandwidth, and the required pump frequency and phase are indicated. And also indicated are the changes in the pass-band arrangement required to improve performance, *e.g.*, in Fig. 7 a narrower idler pass band is needed to increase the bandwidth.

As mentioned above, even though the ω-β curves indicate an ideal pass-band arrangement it will not be possible to obtain gain if the diodes do not couple properly to the fields of the circuit. The coupling of the fields and the diodes can be expressed in terms of three interaction impedances K_s, K_i, and K_p which are the impedances at the signal, idler and pump frequencies, respectively. K is defined by

$$K = \frac{|V_d|^2}{2WV_g}, \tag{4}$$

where

V_d is the peak voltage across the diode,
W is the peak stored electric energy per cavity,
V_g is the group velocity.

Currie and Gould[2] have shown that gain is proportional to the product of the K_s and K_i. The role of K_p is slightly different. A high value of K_p will mean a low-pump power requirement while a low K_p means a large amount of pump power is needed. The value of K_p is not too critical if sufficient pump power is available. For most circuits K varies across the pass bands; this will lead to a variation of gain. In particular, for the circuit presented in this paper, K gets very large near the band edges and since the matching to the circuit gets bad near the band edges a potentially unstable situation exists. However, it is possible in most cases to allow the synchronous conditions [see (1) and (2)] to be far off at the band edges to relieve the unstable condition. Many seemingly complex situations arise when working with TWPA's that can be explained easily in terms of the interaction impedance. An example can best illustrate this point; often as pump power is applied the diodes begin to draw current before significant gain is achieved. This situation cannot be explained in terms of the Brillouin diagram, *i.e.*, the frequency and phase conditions may be optimally met. The solution is simply that K_p is high compared to K_s and/or K_i. A related situation occurs when gain is obtained only when pump power of the order of watts is applied and it would be expected that this large amount of pump power would burn out the diodes. In this case K_p is very low.

Methods of determining the Brillouin diagram and K_s, K_i, and K_p are quite simple. These methods are essentially the same as techniques used in the experimental investigation of circuits for traveling-wave tubes. It is beyond the scope of the paper to discuss the methods in detail, but the major points can be mentioned. For the measurement of the ω-β curve the procedure is the same as for the TWT's, *i.e.*, the circuit is resonated with shorting plates at the input and output, then a perturbation is moved along the circuit to obtain information about the field pattern. For a TWPA, the perturbation used is the change of capacitance of the diodes as the bias is changed. The measurement of K also involves the resonated circuit, but where the diodes are varied successively to determine the ω-β curve, they are all varied simultaneously to find K. For the measurement of K, (4) is more convenient in the form of

$$K = \frac{1}{2} \cdot \frac{1}{\omega_0 V_g} \frac{\Delta\omega}{\Delta C}, \tag{5}$$

where

ω_0 is the resonant frequency,
$\Delta\omega$ is the change of ω_0 in response to a change in C of ΔC. For most purposes, especially with coupled cavity circuits, only $\Delta\omega/\Delta C$ need be noted to determine whether K is large or small.

Conclusion

Development of this device is continuing. Present work is directed toward finding the optimum diode parameters. Difficulty has been experienced with the interchangeability of diodes; this is mostly a mechanical problem, since slight changes in diode position can have large effects. The noise figure is still far above that eventually expected for the amplifier and work will continue to obtain lower values; this is again a problem in finding the optimum diodes; however, the use of higher frequency pass bands for the idler and pump will also be investigated. With the present circuit it is believed that the bandwidths of the order of 50 per cent and noise figures of the order of 2.5 db will be obtained. It is hoped that the present paper will generate new interest in traveling-wave parametric amplifiers.